

MATH2020A Homework 1

7.

$$\begin{aligned}\int_0^1 \int_0^1 \frac{y}{1+xy} dx dy &= \int_0^1 \ln(1+xy) \Big|_0^1 dy = \int_0^1 \ln(1+y) dy \\ &= (y+1) \ln(y+1) - y \Big|_0^1 = 2 \ln 2 - 1\end{aligned}$$

18.

$$\begin{aligned}\iint_R xy e^{xy^2} dA &= \int_0^2 \int_0^1 xy e^{xy^2} dy dx = \int_0^2 \frac{1}{2} e^{xy^2} \Big|_0^1 dx = \int_0^2 \frac{e^x - 1}{2} dx \\ &= \frac{e^x - x}{2} \Big|_0^2 = \frac{e^2 - 3}{2}\end{aligned}$$

21.

$$\begin{aligned}\iint_R \frac{1}{xy} dA &= \int_1^2 \int_1^2 \frac{1}{xy} dy dx = \int_1^2 \frac{1}{x} \ln y \Big|_1^2 dx = \int_1^2 \frac{1}{x} \ln 2 dx \\ &= \ln x \ln 2 \Big|_1^2 = (\ln 2)^2\end{aligned}$$

23.

$$\begin{aligned}\iint_R (x^2 + y^2) dA &= \int_{-1}^1 \int_{-1}^1 (x^2 + y^2) dy dx = \int_{-1}^1 \left(x^2 y + \frac{1}{3} y^3 \right) \Big|_{-1}^1 dx \\ &= \int_{-1}^1 \left(2x^2 + \frac{2}{3} \right) dx = \left(\frac{2}{3} x^3 + \frac{2}{3} x \right) \Big|_{-1}^1 = \frac{8}{3}\end{aligned}$$

25.

$$\begin{aligned}\iint_R (2 - x - y)dA &= \int_0^1 \int_0^1 (2 - x - y)dydx = \int_0^1 \left(2y - xy - \frac{1}{2}y^2\right)\Big|_0^1 dx \\ &= \int_0^1 \left(\frac{3}{2} - x\right)dx = \left(\frac{3}{2}x - \frac{1}{2}x^2\right)\Big|_0^1 = 1\end{aligned}$$